Math 62 13.1-2nd series

Math 72 11.3-1st series

Objectives:
1) Identify finite and infinite series
2) Use summation notation (also called sigma notation)
3) Find partial sums
4) Use and recognize partial sum notation
5) Application problems.

Goal: Add up the terms of a sequence
Add finitely many = finite series
Add infinitely many = infinite series

① The number of baby gorillas born at the San Diego Zoo is a sequence $a_n = n(n-1)$ where n is the number of years the zoo has owned gorillas. Find the total number of baby gorillas born in the first four years.

1st year: $n=1$ $a_1 = 1(1-1) = 0$ gorillas born
2nd year: $n=2$ $a_2 = 2(2-1) = 2$ gorillas born
3rd year: $n=3$ $a_3 = 3(3-1) = 6$ gorillas born
4th year: $n=4$ $a_4 = 4(4-1) = 12$ gorillas born

Total: $0+2+6+12 = \boxed{20 \text{ baby gorillas}}$ in four years

Summation or Sigma Notation is an abbreviation which tells us
- that we are adding terms of a sequence
- the general term of the sequence
- the first value of the index (might not be 1)
- the last value of the index

② Write the series from ① using summation notation.

$$\sum_{i=1}^{4} i(i-1)$$

[OR]

$$\sum_{n=1}^{4} n(n-1)$$

Σ = Greek letter sigma (upper case)

i or n = index variable

1 = starting value, below Σ
4 = ending value, above Σ
$i(i-1)$ or $n(n-1)$ general term.

Math 70

This textbook uses i in chapter 11.3 in the same way that n was used in section 11.1.

The choice of variable is not important, but:

$\sum_{i=1}^{n} a_i$ 1) n is often used to represent the ending value of the index i. If both n and i are used in the same problem, neither can be changed.

$\sum_{n=1}^{3} a_n = \sum_{i=1}^{3} a_i$ 2) if only one index is needed to express a sum, be consistent and use the same letter throughout the problem.

Partial Sum notation

$$S_n = \sum_{i=1}^{n} a_i$$

S_n means the sum of the first n terms. Using partial sum notation assumes the first index value is 1.

(3) Evaluate

a) $\sum_{i=0}^{6} \frac{i-2}{2}$

step 1: substitute $i=0$ $\quad \frac{0-2}{2} = -1$

step 2: substitute $i=1$ $\quad \frac{1-2}{2} = \frac{-1}{2}$

step 3: substitute $i=2$ $\quad \frac{2-2}{2} = 0$

step 4: substitute $i=3$ $\quad \frac{3-2}{2} = \frac{1}{2}$

step 5: substitute $i=4$ $\quad \frac{4-2}{2} = 1$

step 6: substitute $i=5$ $\quad \frac{5-2}{2} = \frac{3}{2}$

step 7: substitute $i=6$ $\quad \frac{6-2}{2} = 2$

step 8: add the results. $\boxed{\frac{7}{2}}$

Tired yet? Do GC worksheet next.

Rev 8-8-13

Name ______________________________

Date ______________________________

TI-84+ GC 37 Finding a Partial Sum, also known as Evaluating a Finite Series

Objective: Use the sum operation from 2nd function LIST to calculate sum of a sequence

LIST will do many things, and we'll use both the sum **sum(** and the sequence **seq(** operations to evaluate a partial sum, also called a finite series.

sum(requires only one input, but it must be a list. **sum(** adds all the items in the list.
seq(requires four inputs:

1. expression to be evaluated,
2. the index variable used in that expression,
3. the starting value of the index variable, and
4. the ending value of the index variable.

These four are listed one after the other, in order, separated by commas.
At the end, we close both sets of parentheses, one for **sum(** and one for **seq(**.

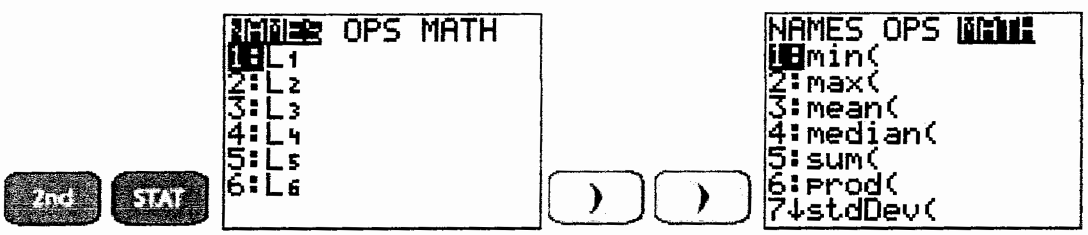
Example 1: What does **sum(seq(**1/n, n, 1,4)) **> frac** do?

Answer: **sum(** adds the the four terms found by **seq(**, then it converts to fraction. $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\boxed{\frac{25}{12}}$

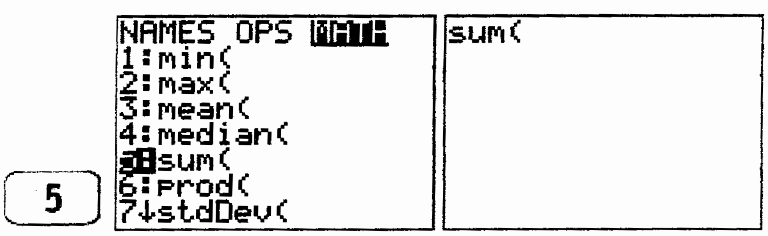
Example 2: Find S_4 when $a_n=\frac{2+3n}{n^2}$.

Note: This same question could be asked using summation or sigma notation:
Find $S_4=\sum_{i=1}^{4}\frac{2+3i}{i^2}$, or just find or evaluate $\sum_{i=1}^{4}\frac{2+3i}{i^2}$.

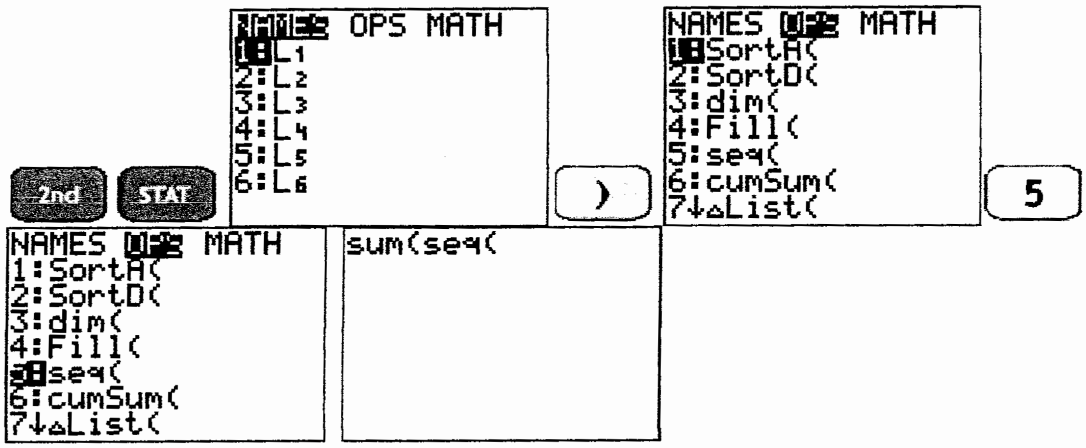
Open LIST, then select right menu MATH:



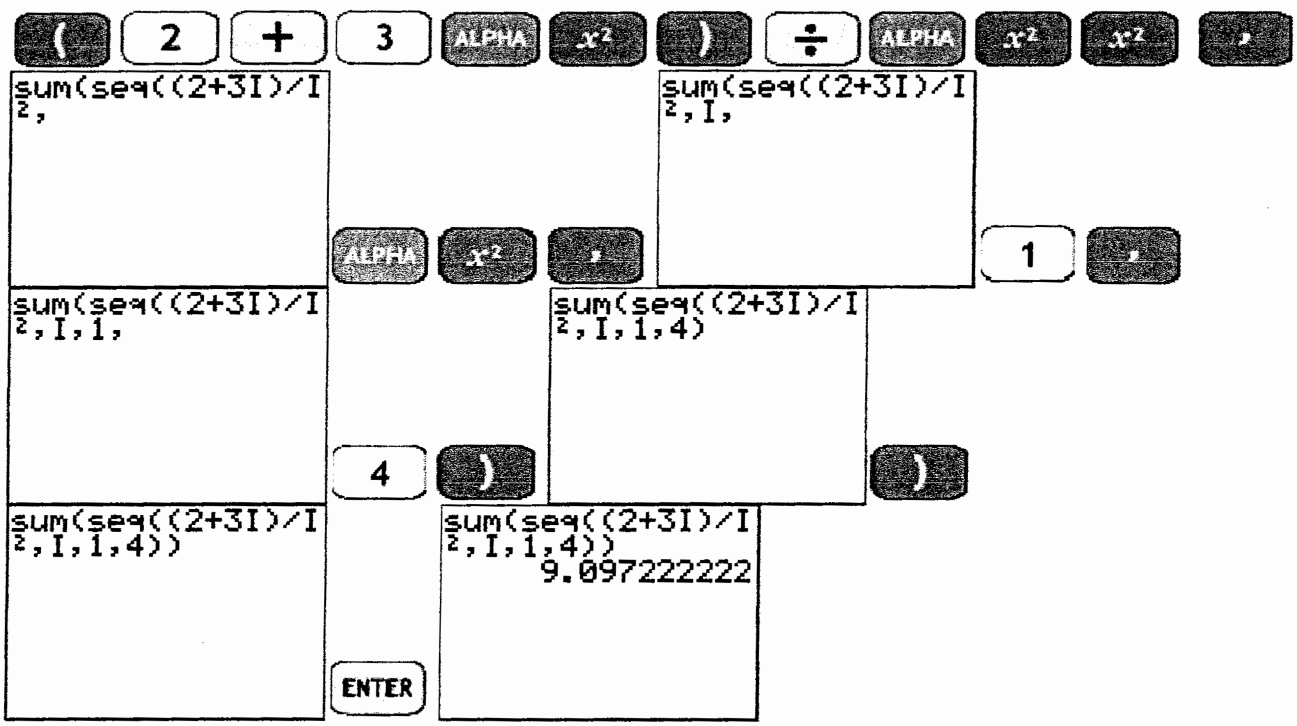
Select option 5 for **sum(**



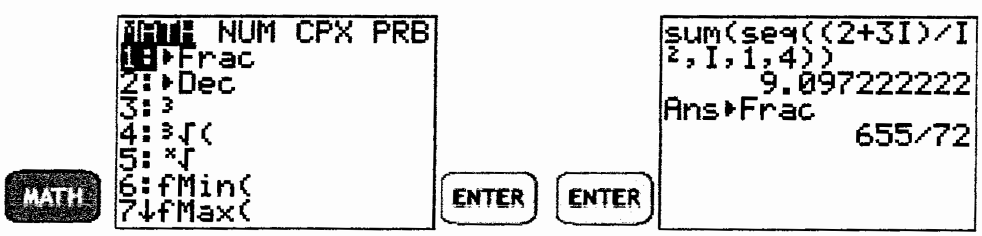
Now we enter **seq(**, which is under the LISTS-OPS menu.



Now we put the four inputs, separated by commas, and close the parentheses twice.



To make this result a fraction (and an exact answer), use MATH > Frac.



Answer: $\boxed{\frac{655}{72}}$

Math 70

3. Continue evaluate:

b) $\sum_{i=3}^{5} 2^i = 2^3 + 2^4 + 2^5 = 8 + 16 + 32 = \boxed{56}$

GC: sum(seq(2^I, I, 3, 6))

c) $\sum_{i=0}^{4} \frac{i-3}{4} = -\frac{3}{4} + \frac{-2}{4} + \frac{-1}{4} + 0 + \frac{1}{4} = \boxed{-\frac{5}{4}}$ or $\boxed{-1.25}$

GC: sum(seq((I−3)/4, I, 0, 4))

d) $\sum_{i=2}^{5} 3^i = 3^2 + 3^3 + 3^4 + 3^5 = \boxed{360}$

GC: sum(seq(3^I, I, 2, 5))

4. Write using summation notation.

yes a) $3 + 6 + 9 + 12 + 15$

$\boxed{\sum_{i=1}^{5} 3i}$

Note: multiples of 3 → add 3 each time means 3 times index, AKA arithmetic sequence. $3 + (i-1)\cdot 3 = 3 + 3i - 3 = 3i$

yes b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

$\boxed{\sum_{i=1}^{4} \frac{1}{2^i}}$

Note: powers of 2 in denom means 2 raised to index geometric sequence $\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)^{i-1} = \left(\frac{1}{2}\right)^i$

no c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} + \frac{1}{14} + \frac{1}{16}$

$\boxed{\sum_{i=1}^{8} \frac{1}{2i}}$

Note: In denom, multiples of 2 means 2 times index neither arithmetic nor geometric

no d) $5 + 10 + 15 + 20 + 25 + 30$

$\boxed{\sum_{i=1}^{6} 5i}$

Note: multiples of 5 means 5 times index arithmetic $5 + (i-1)\cdot 5 = 5 + 5i - 5 = 5i$

no e) $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243}$

$\boxed{\sum_{i=1}^{5} \frac{1}{3^i}}$

Note: Reciprocals of powers of 3. 3 raised to index in denominator geometric sequence $\frac{1}{3}\cdot\left(\frac{1}{3}\right)^{i-1} = \left(\frac{1}{3}\right)^i = \frac{1^i}{3^i} = \frac{1}{3^i}$

Math 70

④ continued.

no f) $-\frac{1}{4}+\frac{1}{8}-\frac{1}{12}+\frac{1}{16}-\frac{1}{20}$

$\boxed{\sum_{i=1}^{5} (-1)^i \cdot \frac{1}{4i}}$ or $\boxed{\sum_{i=1}^{5} \frac{(-1)^i}{4i}}$

Note: alternating sign is power of (-1).
multiple of 4 is 4 times index.
Neither arithmetic nor geometric.

no g) $4+9+16+25+36+49$

$\boxed{\sum_{i=1}^{6} (i+1)^2}$

Note: perfect squares, offset by 1.
neither arithmetic nor geometric

yes h) $2+5+8+11+14$

$$\sum_{i=1}^{5}[2+3(i-1)]$$

$$=\sum_{i=1}^{5} 2+3i-3$$

$$=\boxed{\sum_{i=1}^{5}(3i-1)}$$

Note: add 3 each time means add $3(i-1)$ to a_1
arithmetic sequence

yes i) $\frac{1}{2}+\frac{1}{6}+\frac{1}{18}+\frac{1}{54}$

$$=\boxed{\sum_{i=1}^{4} \frac{1}{2}\left(\frac{1}{3}\right)^{i-1}}$$

Note: multiply by $\frac{1}{3}$ each time means power of $\left(\frac{1}{3}\right)$
$a_1\left(\frac{1}{3}\right)^{i-1}$
geometric sequence

5) Write 4) using partial sum notation.

yes a) S_5 when $a_n = 3n$ is equivalent to $\sum_{i=1}^{5} 3i$ or $\sum_{n=1}^{5} 3n$

no b) S_4 when $a_n = \frac{1}{2^n}$

c) S_8 when $a_n = \frac{1}{2n}$

d) S_6 when $a_n = 5n$

e) S_5 when $a_n = \frac{1}{3^n}$

f) S_5 when $a_n = \frac{(-1)^n}{4n}$

g) S_6 when $a_n = (n+1)^2$

Note: where 4) could have multiple answers (depending on how you offset the index and/or change the starting and ending values of the index), in 5), the index must start with 1.

Extras

Write in summation notation.

6) $10 + 5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8}$

yes

$\boxed{\sum_{i=1}^{5} 5 \cdot \frac{1}{2^{i-2}}}$ or $\boxed{\sum_{i=1}^{5} \frac{5}{2^{i-2}}}$

Note: constant 5 and powers of 2 offset by 2.

geometric sequence $10\left(\frac{1}{2}\right)^{i-1}$

or $\boxed{\sum_{i=1}^{5} 5(2)^{2-i}}$

Since $10 = 5 \cdot 2$

$5 \cdot (2)^1 \cdot \left(\frac{1}{2}\right)^{i-1}$

$= 5(2)^1 \cdot \frac{1}{2^{i-1}}$ → subtract exp $\frac{2^1}{2^{i-1}}$ $1-(i-1)$ $1-i+1$ $2-i$

$= 5(2)^{2-i}$

Math 70

continued

(7) $63 + 21 + 7 + \frac{7}{3} + \frac{7}{9} + \frac{7}{27}$

$$\boxed{\sum_{i=1}^{6} 7 \cdot \frac{1}{3^{i-3}}} = \boxed{\sum_{i=1}^{6} \frac{7}{3^{i-3}}} = \boxed{\sum_{i=1}^{6} 7 \cdot 3^{3-i}}$$

Notes:
Constant 7
powers of 3 in denominator
offset by 3

geometric sequence

$63\left(\frac{1}{3}\right)^{i-1} = 7 \cdot 3^2 \cdot \frac{1}{3^{i-1}} = 7 \cdot 3^{3-i}$

(8) $\frac{8}{3} + \frac{8}{9} + \frac{8}{27} + \frac{8}{81} + \frac{8}{243}$

$$\boxed{\sum_{i=1}^{5} \frac{8}{3^i}} = \boxed{\sum_{i=1}^{5} \frac{8}{3}\left(\frac{1}{3}\right)^{i-1}}$$

Note: constant 8
Powers of 3 in denominator
geometric sequence $\frac{8}{3}\left(\frac{1}{3}\right)^{i-1}$

(9) $\frac{100}{7} + \frac{10}{7} + \frac{1}{7} + \frac{1}{70} + \frac{1}{700} + \frac{1}{7000}$

$$\boxed{\sum_{i=1}^{6} \frac{1}{7} \cdot \frac{1}{10^{i-3}}} = \boxed{\sum_{i=1}^{6} \frac{1}{7}(10)^{3-i}}$$

Note: constant $\frac{1}{7}$
powers of 10 in denominator
offset by 3

geometric sequence

$\frac{100}{7}\left(\frac{1}{10}\right)^{i-1} = \frac{10^2}{7} \cdot 10^{1-i} = \frac{1}{7}(10)^{3-i}$

(10) $2 + 6 + 10 + 14 + 18 + 22 + 26$

$$\sum_{i=1}^{7} 2 + 4(i-1)$$

$$\sum_{i=1}^{7} 2 + 4i - 4 \qquad \text{distribute}$$

$$\boxed{\sum_{i=1}^{7} 4i - 2} \qquad \text{combine like terms.}$$

Note: Add 4 each time
start offset by 2

arithmetic sequence

(11) $1 + 4 + 9 + 16 + 25 + 36 + 49$

$$\boxed{\sum_{i=1}^{7} i^2}$$

neither arithmetic
nor geometric

11.3.21 **Write the series with summation notation.**

$4+8+16+32$

Let the sum start with i = 1. Where does the summation end?

$$\sum_{i=1}^{4}$$

What is the general term for the summation?

$$\sum_{i=1}^{4} 4\cdot 2^{i-1}$$

Since we multiply by 2 to get from

4 to 8

8 to 16

16 to 32

The general term has 2^i in it.

We need to make two adjustments, potentially

1) Multiply by the first term (to make MXL happy)

2) Adjust the index, if necessary.

$4\cdot 2^i$ Test $i=1 \Rightarrow 4\cdot 2^1 = 8$ is a term in the sequence, but not the first $\rightarrow$ it's one to the right.

We want the answer 8 to result when $i=2$.

Try again, adjusting by 1.

$4\cdot 2^{i-1}$

Test $i=1 \Rightarrow 4\cdot 2^{1-1} = 4\cdot 1 = 4$ ✓

Test $i=2 \Rightarrow 4\cdot 2^{2-1} = 4\cdot 2 = 8$ ✓

Test $i=3 \Rightarrow 4\cdot 2^{3-1} = 4\cdot 2^2 = 16$ ✓.

$$\boxed{\sum_{i=1}^{4} 4\cdot 2^{i-1}}$$

11.3.49

A pendulum swings a length of 38 inches on its first swing. Each successive swing is $\frac{9}{10}$ of the preceding swing. Find the length of the sixth swing and the total length swung during the first six swings.

The length of the sixth swing is ☐ in.
(Round to the nearest tenth as needed.)

The total length swung during the first six swings is ☐ in.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

1st swing 38 $\quad N=1 \quad 38\cdot\left(\frac{9}{10}\right)^{1-1}$

2nd $\frac{9}{10}(38) = \frac{171}{5}$ $\quad N=2 \quad 38\cdot\left(\frac{9}{10}\right)^{2-1}$

3rd $\frac{9}{10}\left(\frac{171}{5}\right) = \frac{1539}{5}$

4th $\frac{9}{10}\left(\frac{1539}{5}\right) = \frac{13851}{500}$

5th $\frac{124659}{5000}$

6th 22.43862 $\quad \boxed{\text{Nearest tenth } 22.4}$

General term: $38\cdot\left(\frac{9}{10}\right)^{N-1}$

$$\text{sum}\left(\text{Seq}\left(38\cdot\left(\frac{9}{10}\right)^{N-1}, N, 1, 6\right)\right) = 178.05242$$

$\boxed{\text{Nearest tenth } 178.1}$

$a_n = a_0 + (n-1)d$ arithmetic

$a_n = a_1 \cdot r^{n-1}$ geometric